Mid-Term Revision

(I) Proofs:

1. Describe the principle of energy Conversion, Develop the model of electro-mechanical energy Conversion device

   Pages: (2-2, 2-3, 2-4)

2. Derive an expression for mechanical work done in Case of fast & slow motions

   Pages: (2-9, 2-10, 2-11, 2-12)

3. Derive an expression for the Mechanical force produced by single excited magnetic relay

   Pages: (2-13, 2-14)

4. Derive an Expression for the torque in the reluctance motor & state the Conditions for non-zero average torque.
   Pages: (2-34, 35, 36, 39) (Electro magnetic Synchronous Motor)

5 Find the total energy stored in the magnetic field for multi-excited magnetic system

Pages : (2 - 46, 47)

6 Derive an expression for the force in double excited system (V.I)

Pages : (2 - 47, 48)

7 Derive an expression for the torque in double excited system (rotating) (V.I)

Pages: (2-47, 48)

8 Derive an expression for the force in electro-static system (2- 68, 69)

9) Derive an expression For Torque in Electro Static Synchronous Motor.

(2-2)

- Energy balance eqn:-

Total i/p energy = Total energy stored + total energy loss (dissipated).

where;

→ Total i/p energy = Electrical i/p energy (wei) + mechanical i/p energy (Wmi).

→ Total energy stored = Mech. energy stored (Wms) + elec. energy stored (Wes)

magnetic field or electric field

→ Total energy dissipated = Mech. loss (Wml) + electric loss (Wel)

- ohmic losses
- field losses

→ In mathematical form:-

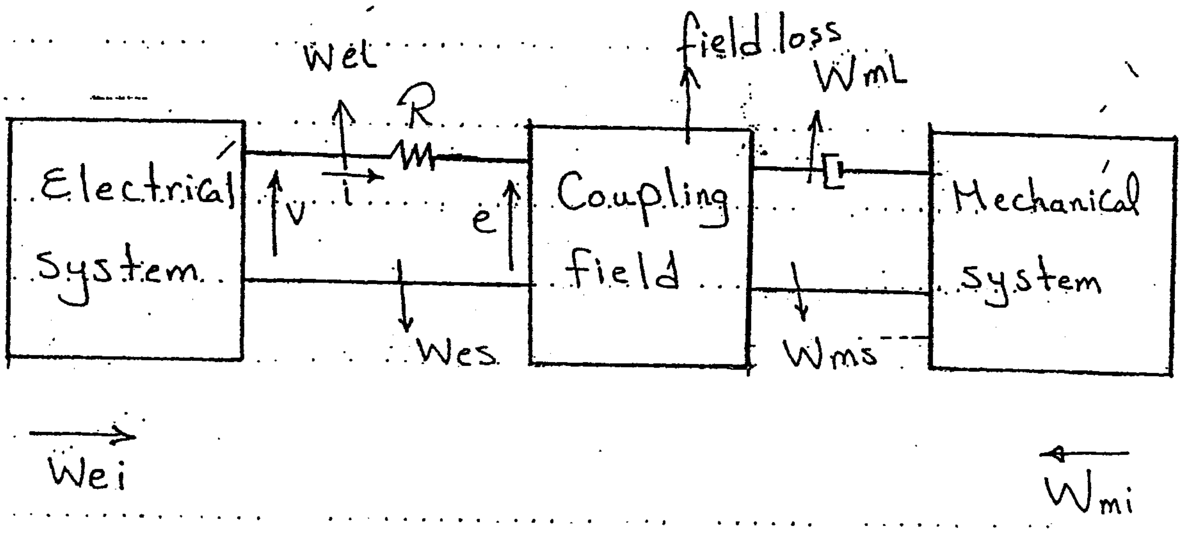
$$\boxed{W_{ei} + W_{mi} = (W_{es} + W_{ms}) + (W_{el} + W_{ml})}$$

→ In differential form:-

$$\boxed{dW_{ei} + dW_{mi} = dW_{es} + dW_{ms} + dW_{el} + dW_{ml}}$$

(2-3)

→ The electromechanical energy conversion model can be represented by:-



→ Notes:-

we can study two applications :-

(1) Motor :-

$$\boxed{dw_{mi} = -dw_{mo}}$$

$\therefore dw_{ei} - dw_{mo} = dw_{es} + dw_{ms} + dw_{el} + dw_{mL}$

$\therefore dw_{ei} = dw_{mo} + dw_{es} + dw_{ms} + dw_{mL}$ + ohmic losses + field loss.

$\therefore dw_{ei}$ − Cu loss (ohmic) $= dw_{mo} + dw_{mL} + dw_{es} + dw_{ms}$ + field loss.

$$\therefore \boxed{dW_{elec} = dW_{mech} + dW_{field}}$$

dW_{elec} = dw_{ei} − ohmic loss

$dW_{mech} = dw_{mo} + dw_{mL}$

$dW_{field} = dw_{es} + dw_{ms}$ + field loss

(2-4)

(2) Generator :-

$$\therefore dW_{ei} = -dW_{eo}.$$

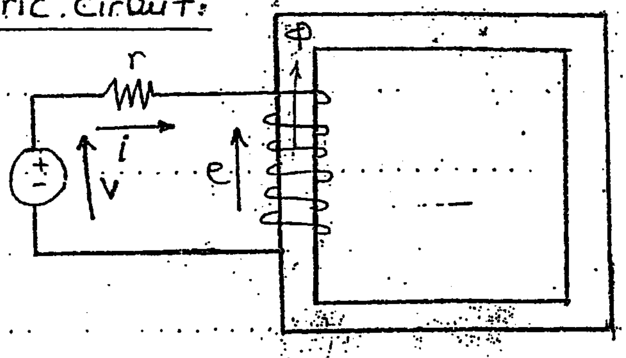
$$\therefore dW_{mi} - dW_{eo} = dW_{es} + dW_{ms} + \text{ohmic loss} + \text{field loss} + dW_{ml}.$$

$$\therefore \underbrace{dW_{mi} - dW_{ml}}_{dW_{mech}} = \underbrace{dW_{eo} + \text{Cu loss}}_{dW_{elec}} + \underbrace{dW_{es} + dW_{ms} + \text{field loss}}_{dW_{field}}$$

$$\therefore \boxed{dW_{mech} = dW_{elec} + dW_{field}}$$

III) Single excited magnetic system :-

• Apply KVL on electric circuit:



$$\therefore V = ir + e ; \left(e = N\frac{d\phi}{dt}\right)$$

$$\therefore V = ir + N\frac{d\phi}{dt}$$

$$\because \lambda = N\phi$$

$$\therefore V = ir + \frac{d\lambda}{dt}$$

multiply both by $(i\,dt)$.

$$\therefore V i\,dt = i^2 r\,dt + i\,d\lambda.$$

(i/p elec. energy) dW_{ei} ; $i^2 r\,dt$ → ohmic loss ; $i\,d\lambda$ → dW_{fld} (energy stored)

$$\therefore dW_{ei} - d(\text{ohmic loss}) = dW_{fld}$$

$$\therefore \boxed{dW_{elec} = dW_{fld}} = i\,d\lambda$$

(2-9)

(3) → During Motion:-

. The arm Can move by:-

(1) Slow motion:-

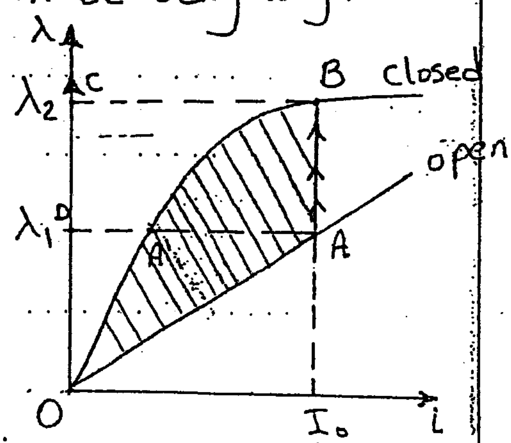
- the time taken for motion will be very long.

$\therefore dt \uparrow\uparrow , e = \frac{d\lambda}{dt}$

$\therefore e = 0$

@ this Case: $i = I_o$

$\therefore$ the Current is Const.



. Applying energy balance eqn:-

$\therefore \Delta W_{elec} = \Delta W_{mech} + \Delta W_{field}$.

$\because$ I is Const.

$\therefore$ the path of motion from (A) to (B) is the vertical line @ $i = I_o$.

$$\therefore \Delta W_{elec} = \int_{\lambda_1}^{\lambda_2} i \, d\lambda = \int_{\lambda_1}^{\lambda_2} I_o \, d\lambda$$

$= \text{Area } (ABCDA'A)$

$$\therefore \Delta W_{field} = W_{field}\Big|_{closed} - W_{field}\Big|_{opened}.$$

$= A(OBCDO) - A(OAA'DO)$.

$\therefore$ from energy balance eqn. :-

$$\Delta W_{mech} = \Delta W_{elec.} - \Delta W_{field.}$$

$$\therefore \Delta W_{mech} = \text{Area}\,[ABCDA'A - OBCDO + OAA'DO]$$
$$= \text{Area}\,[OABA'O].$$

$$\boxed{\therefore F_e\Big|_{average} = \frac{\Delta W_{mech}}{g}}$$

where;

$F_e\Big|_{average}$: Average electro magnetic force.

g : Gap distance.

N.B.:-

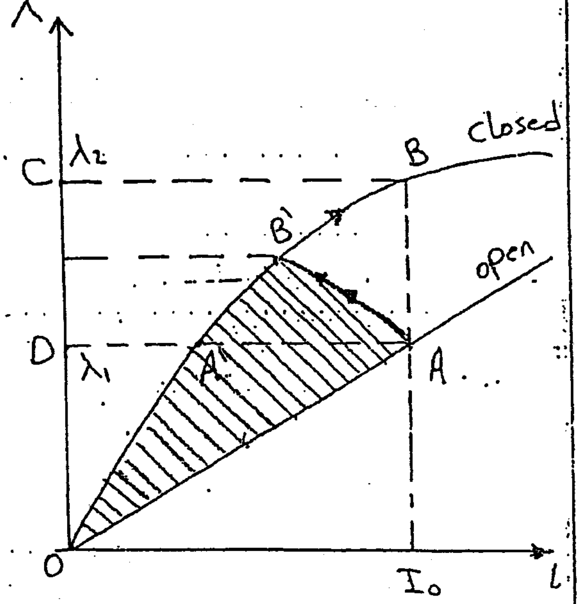
1) ΔW_{mech} : Area btn path of motion & the two Curves (open & closed).

2) ΔW_{elec} : Area btn path of motion & the vertical axis.

$$\therefore F_e \Big|_{average} = \frac{\Delta W_{mech}}{g}$$

Also; ΔW_{mech} = area btn path of motion & the two Curves C/Cs.

(3) Intermediate motion:- (General Case).

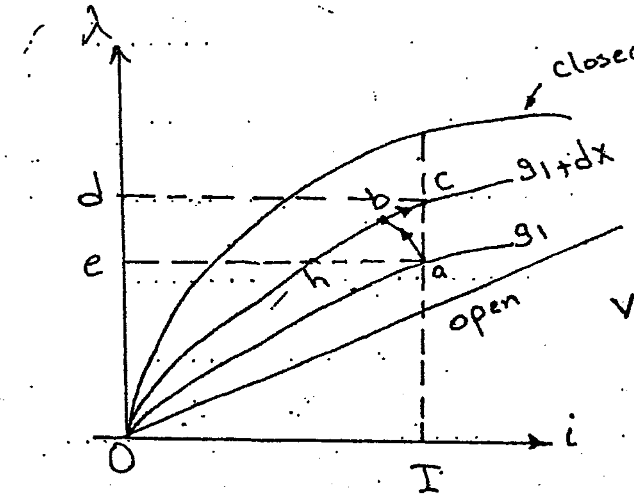


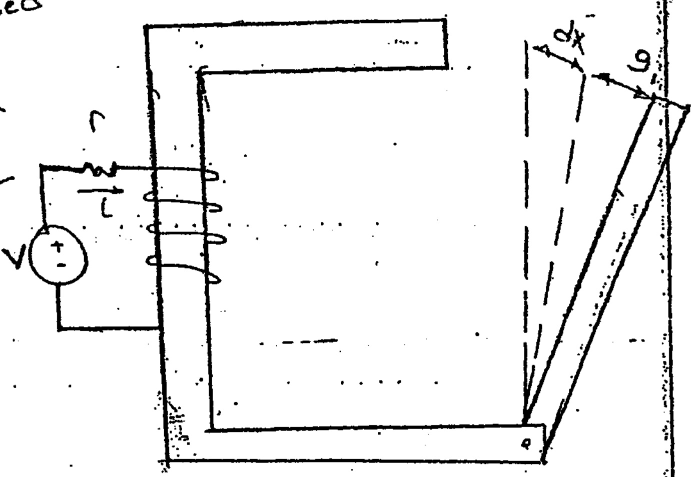
As discussed before:-

- $\Delta W_{mech} = A(OAB'A'O)$.
  = Area btn path of motion & the two C/Cs.
- path of motion is AB'.
- $F_e \Big|_{avg} = \dfrac{\Delta W_{mech}}{\text{gap distance } (g)}$

→ let us get an expression for the instantaneous force as a function of the position.

V) Instantaneous force:- (Mathematical Solution)





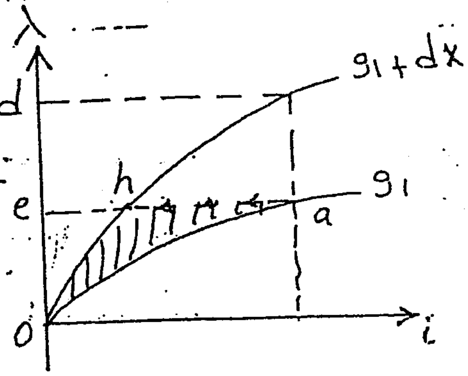
- Assume that the motion is from $g_1 \longrightarrow g_1 + dx$.

  $\therefore$ dW_{mech} = Area btn path of motion & two curves.

  $\therefore$ $dW_{mech} = A(Oabho) = f_e\,dx$.

- we can have two assumptions:-

(1) Neglect area (abh):-



- It looks like fast motion Case.

$\therefore$ $\lambda = $ Const.

$\therefore$ $dW_{mech} = f_e\,dx$.

$\therefore$ $dW_{elec} = 0$ ($\lambda = $ Const).

$\therefore$ $0 = dW_{mech} + dW_{field}$ (balance eqn).

$\therefore$ $dW_{mech} = -dW_{field} = f_e\,dx$.

for linear system $\rightarrow$ $\therefore$ $\boxed{f_e = -\left.\dfrac{dW_{field}}{dx}\right|_{\lambda = \text{Const.}}}$

&

for nonlinear system $\longrightarrow$ $\therefore$ $\boxed{f_e = -\dfrac{\partial W_{field}(\lambda, x)}{\partial x}}$

(1-14)

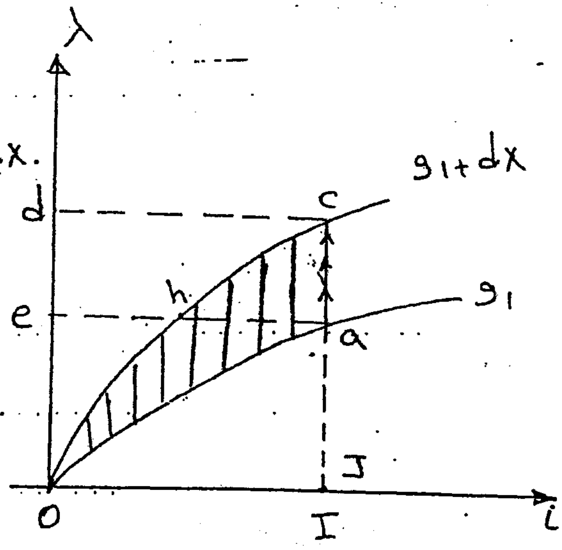
(2) Add area (abc):-

- Like slow motion case.

$dW_{mech} = Area(0ach0) = f_e\,dx.$

$\therefore A(0ach0) = A(0hcaJ0) - A(0aJ0).$

$= W'_{field}\Big|_{g_1+dx} - W'_{field}\Big|_{g_1}$



$\therefore dW_{mech} = dW'_{field} = f_e\,dx.$

$\therefore$ for Linear system:-

$$\boxed{f_e = \frac{dW'_{field}}{dx}\Bigg|_{i=Const.}}$$

$\therefore$ for non-linear system:-

$$\boxed{f_e = \frac{\partial W'_{field}(i,x)}{\partial x}}$$

N.B:- In linear system:-

1) $f_e = -\frac{dW_{field}}{dx}\Big|_{\lambda=Cons}$, $W_{field} = \frac{1}{2}\phi^2 R.$

$$\boxed{\therefore f_e = -\frac{1}{2}\phi^2\frac{dR}{dx}}$$

(2-28)

IV) Single excited rotating system:-

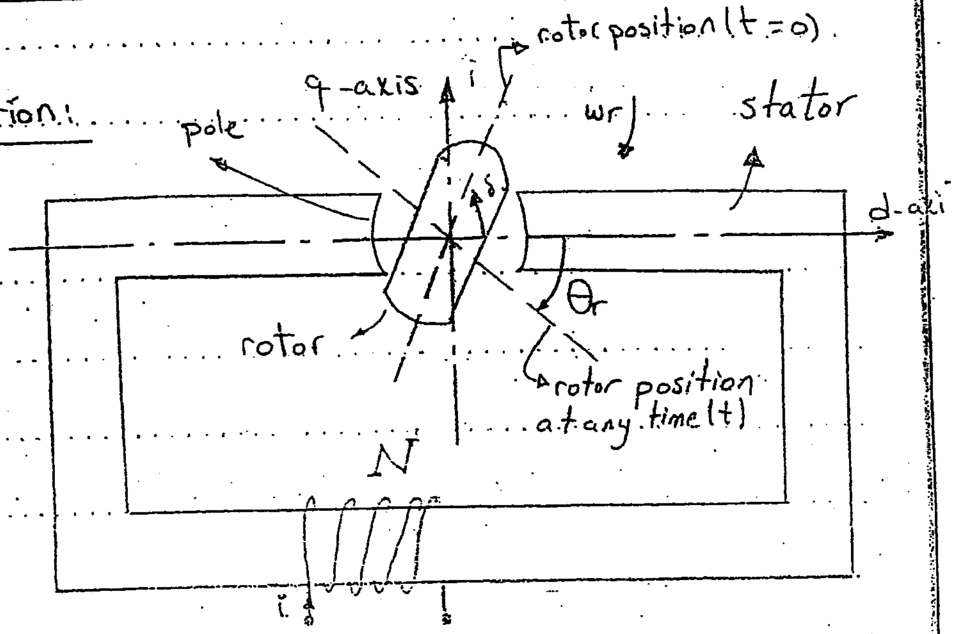
- There is analogy btn linear translational system & rotational system as follows:-

Translational	Rotational
Force (Fe)	Torque (Te)
Displacement (X)	angular position (θ)
speed (V)	Angular speed (ω)

- Now, we are going to study an important application for single excited rotating system which is the "Reluctance Motor"

• Reluctance Motor:-

(i) Construction:



- The reluctance motor consists of three main parts:-

(1) Stator : fixed body where a coil is placed on & it is made of magnetic material.

(2) Rotor : The rotating part of the motor, where the shaft is found on it.

(3) Airgap:- the clearance btn the stator & the rotor where electro_mechanical energy conversion takes place.

(ii) Definitions:

(1) θr : Rotor angle measured from d-axis.

(2) δ : Initial position of rotor measured from d-axis

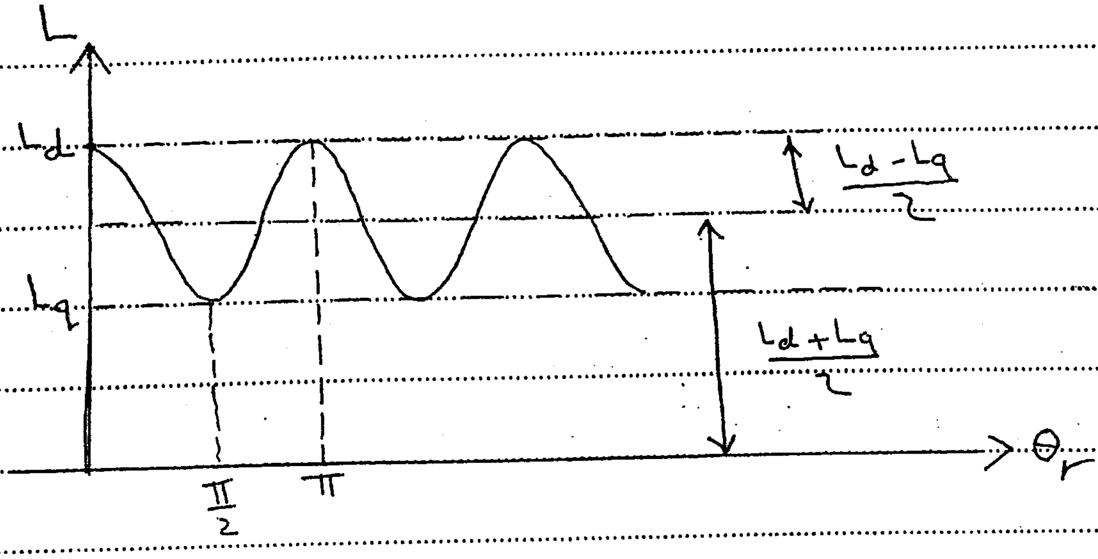
(3) wr : Rotor angular speed (rad/s).

- from the figure: $\boxed{\theta r = wrt - \delta}$

- Using $wr = \frac{d\theta r}{dt} \Rightarrow \boxed{wr = \dot{\theta}}$, $\boxed{wr = \frac{2\pi\, nr}{60}}$

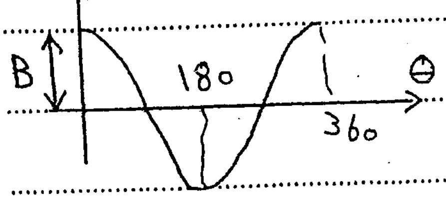
where : nr = speed in (rpm)

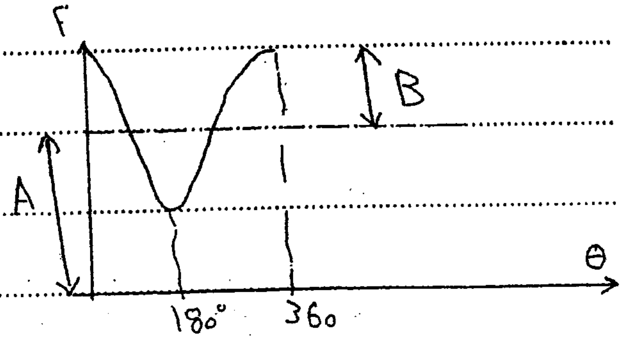
* Relation between The inductance & θ_r :



$$L(\theta_r) = \frac{L_d + L_q}{2} + \frac{L_d - L_q}{2} \cos 2\theta_r$$

Note :





$F = B \cos\theta$

$F = A + B \cos\theta$

If:

$$L_d = \frac{N^2}{R_d} \quad \& \quad L_q = \frac{N^2}{R_q}$$

$$\therefore \boxed{L = \frac{1}{2}(L_d + L_q) + \frac{1}{2}(L_d - L_q)\cos(2\theta_r)} \quad (\text{Rep})$$

(iv) Torque equation:-

- We have two methods to find the torque (assuming Linear system) using W_{fld} & W'_{fld}.

- The stator is assumed to have infinite permeability, so it will have zero reluctance & the only considered reluctance is the air gap reluctance which we calculated in the last papers.

- for rotational systems:

$$T_e = \frac{1}{2} i^2 \frac{dL}{d\theta_r} = \frac{dW'_{fld}}{d\theta_r}$$

&

$$T_e = -\frac{1}{2}\phi^2 \frac{dR}{d\theta_r} = -\frac{dW_{fld}}{d\theta_r}$$

(2-34)

(1) Torque in terms of motor current (i):

• Using:

$$T_e = \frac{1}{2} i^2 \frac{dL}{d\theta_r}$$

• And, the current (i) should be (a.c) sinusoidal & is given by:

$$i = I_m \cos\omega t \qquad (\omega = 2\pi f)$$

or $$i = I_m \sin\omega t \qquad (\omega = 2\pi f)$$

• But, we will consider:

$$i = I_m \cos\omega t$$

Proof: (very important)

$$\because L(\theta_r) = \frac{1}{2}(L_d + L_q) + \frac{1}{2}(L_d - L_q)\cos(2\theta_r)$$

$$\therefore \frac{dL}{d\theta_r} = 0 + \frac{1}{2}(L_d - L_q)(2)(-\sin 2\theta_r)$$

$$\because i = I_m \cos\omega t \;\&\; T_e = \frac{1}{2} i^2 \frac{dL}{d\theta_r}$$

$$\therefore T_e = \frac{1}{2}(I_m^2 \cos^2\omega t)\left(\frac{1}{2}(L_d - L_q)(2)(-\sin 2\theta_r)\right)$$

$$\therefore \boxed{T_e = -\frac{1}{2} I_m^2 \cos^2\omega t\,(L_d - L_q)\sin(2\theta_r)}$$

(2-35)

- The previous eqn is the torque as a fn of time (t) & Called "instantenous torque"
- for uni-directional rotation we should have non-zero average torque

- So, we have to calculate the average torque:

$$\because \cos^2 \omega t = \frac{1}{2}(1 + \cos 2\omega t).$$

$$\therefore T_e = -\frac{1}{4} I_m^2 (L_d - L_q)(1 + \cos 2\omega t) \sin(2\theta_r).$$

$$\therefore T_e = -\frac{1}{4} I_m^2 (L_d - L_q).(\sin(2\theta_r) + \sin(2\theta_r) \cos 2\omega t)$$

But we know that:

$$\sin A . \cos B = \frac{1}{2}(\sin(A+B) + \sin(A-B))$$

$$\therefore T_e = -\frac{1}{4} I_m^2 (L_d - L_q)\left[\sin 2\theta_r + \frac{1}{2}\sin(2\theta_r + 2\omega t) + \frac{1}{2}\sin(2\theta_r - 2\omega t)\right]$$

But, $\theta_r = w_r t - \delta$

$$\therefore T_e = -\frac{1}{4} I_m^2 (L_d - L_q)\left[\sin 2(w_r t - \delta) + \frac{1}{2}\sin(2w_r t + 2\omega t - 2\delta) + \frac{1}{2}\sin(2w_r t - 2\omega t - 2\delta)\right]$$

(2-36)

- We have three terms:-

  * $\sin 2(w_r t - \delta)$ $\Rightarrow$ have zero average (sinusoidal).

  * $\sin(2(w + w_r)t - 2\delta)$ $\Rightarrow$ have zero average (sinusoidal)

  * $\sin(2w_r t - 2wt - 2\delta)$ $\Rightarrow$ have two cases

Case(1):-

if $w_r \neq w$ $\Rightarrow$ $\sin(2w_r t - 2wt - 2\delta)$ will have zero average (sinusoidal also).

Case(2):

if $w_r = w$ $\Rightarrow$ $\sin(2\cancel{w_r} t - 2\cancel{w}t - 2\delta) = -\sin(2\delta)$

$\therefore \sin(-2\delta) =$ non-zero value

$\therefore$ The Torque (T_e) will have non-zero average torque if:

$$\boxed{w_r = w}$$

Now, the average torque ($w_r = w$) will be given by:-

$$\boxed{T_e \Big|_{avg} = \frac{1}{8} I_m^2 (L_d - L_q) \sin 2\delta}$$

(2-39)

• Important Notes:-

(1) Conditions for non-zero torque:-

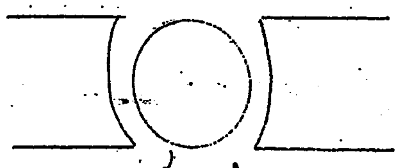
(i) $\omega r = \omega = 2\pi f$ (Proved).

(ii) $\delta \neq$ zero ($Te|_{avg} \propto \sin 2\delta$).

(iii) $Rd \neq Rq$.

or

$Ld \neq Lq$



(ylinderical rotor)

$\therefore$ $\boxed{\text{The rotor should not be Cylinderical.}}$

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(2) The maximum average torque occurs at $\delta = 45°$.

$\because Te|_{avg} \propto \sin 2\delta \Rightarrow \delta = 45°$ (for max. value)

$$\therefore Te|_{avg} = \frac{1}{8} I_m^2 (Ld - Lq).$$

$$\& \; Te|_{avg} = \frac{1}{8} \Phi_m^2 (Rq - Rd).$$

+6+7

(2-46)

• **flux linkage:-**

$$\boxed{\begin{array}{ll} \lambda_1 = l_{11} i_1 + M i_2 & \text{flux linkage in Coil(1)} \\ \lambda_2 = l_{22} i_2 + M i_1 & \text{flux linkage in Coil(2)} \end{array}}$$

where;

$l_{11} \equiv$ self inductance of Coil (1).

$l_{22} \equiv$ " " " " (2).

M or $l_{12} \equiv l_{21} \equiv$ Mutual inductance btn Coils (1) & (2).

* **Expressions for energy & Co-energy :-** (V.I)

→ Consider no motion :-

• Apply the energy balance eqn :

$\because dw_{elec} = dw_{mech} + dw_{fld}$

$\therefore dw_{elec} = dw_{fld}$ ($dw_{mech} = 0$).

$\therefore dw_{fld} = i_1 d\lambda_1 + i_2 d\lambda_2$ (two Coils).

$\because \lambda_1 = l_{11} i_1 + M i_2$.

& $\lambda_2 = l_{22} i_2 + M i_2$.

$\therefore dw_{fld} = i_1 (d(l_{11} i_1 + M i_2)) + i_2 (d(l_{22} i_2 + M i_1))$

$\therefore dw_{fld} = l_{11} i_1 di_1 + M i_1 di_2 + M i_2 di_1 + i_2 l_{22} di_2$.

$= l_{11} i_1 di_1 + i_2 l_{22} di_2 + M\, d i_1 i_2$.

N.B:- l_{11}, l_{22} & M are Considered Constant as there is no motion.

(2-47)

$$\therefore W_{fld} = \int dW_{fld}.$$

$$\boxed{\therefore W_{fld} = \frac{1}{2} L_{11} i_1^2 + \frac{1}{2} L_{22} i_2^2 + M i_1 i_2.}$$

→ In linear systems:-

$$\therefore W_{fld} = W'_{fld} = \frac{1}{2} L_{11} i_1^2 + \frac{1}{2} L_{22} i_2^2 + M i_1 i_2.$$

$\therefore$ for n- Coil linear system:-

$$\boxed{W_{fld} = W'_{fld} = \sum_{j=1}^{n} \sum_{K=1}^{n} \frac{1}{2} L_{jK}\, i_j i_K.}$$

* Expression for instantaneous force & torque:-

→ from Energy balance eqn:-

$$\therefore dW_{elec} = dW_{mech} + dW_{fld}.$$

$\because$ L_{11}, L_{22} & M are not Constants as the system is in motion.

$$\therefore dW_{elec} = i_1 d\lambda_1 + i_2 d\lambda_2.$$

$$= L_{11} i_1 di_1 + i_1^2 dL_{11} + M i_1 di_2 + i_1 i_2 dM$$
$$+ M i_2 di_1 + i_1 i_2 dM + L_{22} i_2 di_2 + i_2^2 dL_{22}$$

(2-48)

$$\therefore dW_{elec} = L_{11} i_1 di_1 + L_{22} i_2 di_2 + i_1^2 dL_{11} + i_2^2 dL_{22} + M(i_1 di_2 + i_2 di_1) + 2 i_1 i_2 dM \longrightarrow (1)$$

$$\because W_{fld} = \frac{1}{2} i_1^2 L_{11} + \frac{1}{2} i_2^2 L_{22} + i_1 i_2 M.$$

$$\therefore dW_{fld} = L_{11} i_1 di_1 + \frac{1}{2} i_1^2 dL_{11} + L_{22} i_2 di_2 + \frac{1}{2} i_2^2 dL_{22} + M i_2 di_1 + M i_1 di_2 + i_1 i_2 dM.$$

$$\therefore dW_{fld} = L_{11} i_1 di_1 + L_{22} i_2 di_2 + \frac{1}{2} i_1^2 dL_{11} + \frac{1}{2} i_2^2 dL_{22} + M(i_1 di_2 + i_2 di_1) + i_1 i_2 dM. \longrightarrow (2)$$

$$\because dW_{mech} = dW_{elec} - dW_{fld}$$

- from (1) & (2) :-

$$\therefore dW_{mech} = \frac{1}{2} i_1^2 dL_{11} + \frac{1}{2} i_2^2 dL_{22} + i_1 i_2 dM.$$

→ for translational system:- $\left(F_e = \frac{dW_{mech}}{dx} \right)$.

$$\boxed{\therefore F_e = \frac{1}{2} i_1^2 \frac{dL_{11}}{dx} + \frac{1}{2} i_2^2 \frac{dL_{22}}{dx} + i_1 i_2 \frac{dM}{dx}}$$

→ for rotational system:- $\left(T_e = \frac{dW_{mech}}{d\theta} \right)$.

$$\boxed{\therefore T_e = \frac{1}{2} i_1^2 \frac{dL_{11}}{d\theta} + \frac{1}{2} i_2^2 \frac{dL_{22}}{d\theta} + i_1 i_2 \frac{dM}{d\theta}}$$

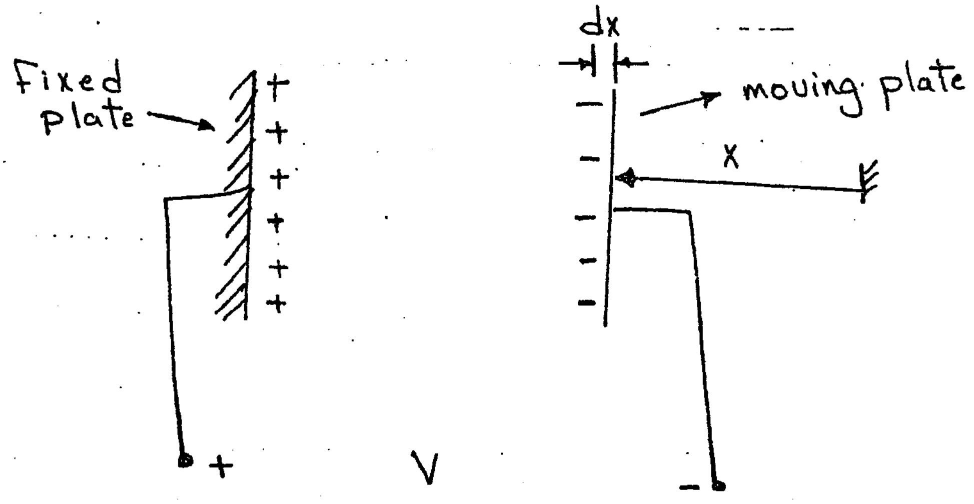
(2-49)

→ for n_Coils :-

$$F_e = \sum_{j=1}^{n}\sum_{k=1}^{n} \frac{1}{2} i_j i_k \frac{dL_{jk}}{dx} \quad \text{(translational)}$$

$$T_e = \sum_{j=1}^{n}\sum_{k=1}^{n} \frac{1}{2} i_j i_k \frac{dL_{jk}}{d\theta} \quad \text{(rotational)}.$$

* Expression for electrostatic force :- (2-68)



$$\because \ dw_{elec} = dW_{mech} + dW_{fld}.$$

$$\because \ dw_{elec} = V\,dq \quad \& \quad q = CV.$$

$$\therefore \ dw_{elec} = V\,d(CV) = V^2 dC + CV\,dV$$

$$\because \ W_{fld} = \tfrac{1}{2} qV = \tfrac{1}{2} CV^2.$$

$$\therefore \ dW_{fld} = CV\,dV + \tfrac{1}{2} V^2 dC.$$

$$\therefore \ V^2 dC + CV\,dV = CV\,dV + \tfrac{1}{2} V^2 dC + dW_{mech}.$$

$$\boxed{\therefore \ dW_{mech} = \tfrac{1}{2} V^2 dC. \quad \& \quad \therefore \ F_e = \tfrac{1}{2} V^2 \frac{dC}{dx}}$$

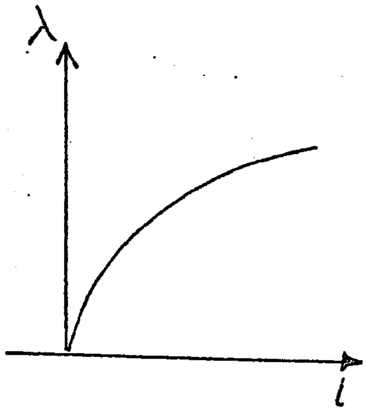
(2-69)

- In terms of (q), Fe will be

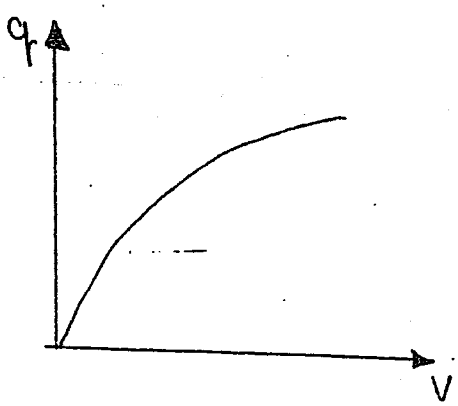
$$\boxed{\therefore \; F_e = -\frac{1}{2} q^2 \frac{d}{dx}\left(\frac{1}{C}\right)}$$

- from the previous eqns & Comparing with electro-magnetic system:

Electromagnetic		electrostatic
B	$\longrightarrow$	D
λ	$\longrightarrow$	q
i	$\longrightarrow$	V
L	$\longrightarrow$	C
μ	$\longrightarrow$	ϵ



(λ-i) C/C's



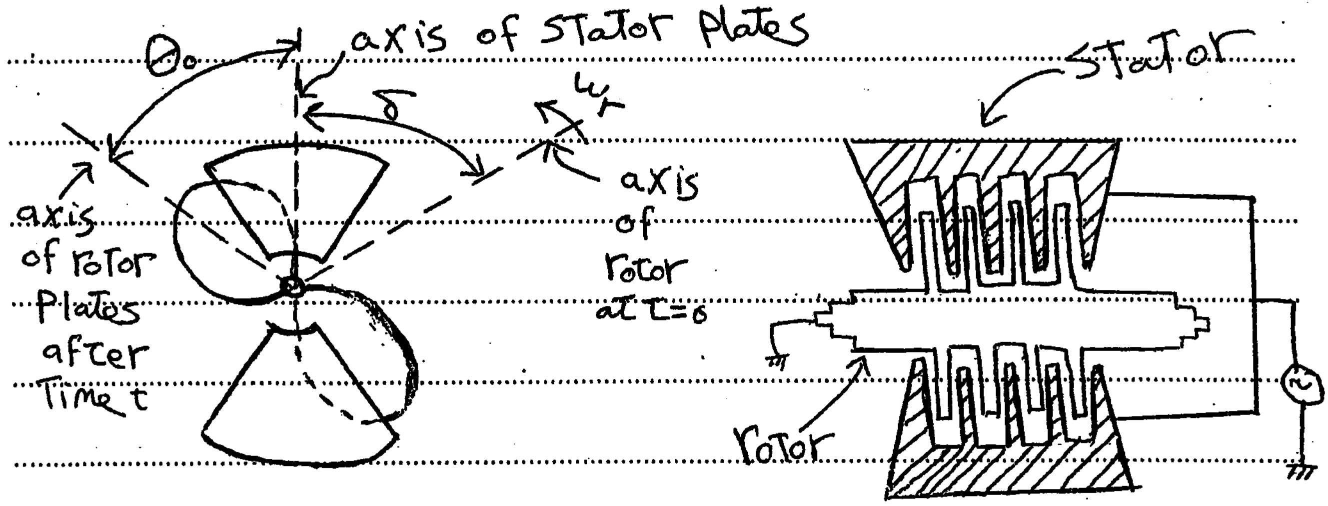
(q-v) C/C's.

Single phase Electrostatic Synchronous Machine:

(V.I)

* An electrostatic synchronous M/c is analogous to magnetic reluctance Motor.

* The plates are shaped to make "C" varies sinusoidal with θ.



$$C = \frac{C_{max} + C_{min}}{2} + \frac{C_{max} - C_{min}}{2} \cos 2\theta_o$$

where: $\theta_o = \omega_r t - \delta$

$$T = \frac{1}{2} V^2 \frac{dC}{d\theta_r} \quad ; \quad V = V_m \cos \omega t$$

The same proof of reluctance Motor

$$\boxed{T_{avg} = \frac{1}{8} V_{max}^2 (C_{max} - C_{min}) \sin 2\delta}$$

جيد

Pb (7) : Mid Term 2009, 2012, Final 2015

Given $\lambda_1 = x^2 i_1^2 + x\, i_2$

$\lambda_2 = x^2 i_2^2 + x\, i_1$

Find w_{Fld}, w'_{Fld} & w_{mech} if $X : 0 \rightarrow 5$ cm

Solution:

$$w'_{Fld} = \int \lambda_1 di_1 + \lambda_2 di_2 = \int (x^2 i_1^2 + x\, i_2)\, di_1 + (x^2 i_2^2 + x\, i_1)\, di_2$$

$$= \frac{x^2 i_1^3}{3} + \frac{x^2 i_2^3}{3} + x\, i_1 i_2$$

$$\therefore w_{Fld} + w'_{Fld} = i_1 \lambda_1 + i_2 \lambda_2 = x^2 i_1^3 + x\, i_1 i_2 + x^2 i_2^3 + x\, i_1 i_2$$

$$\therefore w_{Fld} = \frac{2}{3} x^2 i_1^3 + \frac{2}{3} x^2 i_2^3 + x\, i_1 i_2$$

$$F_e = \frac{\partial w'_{Fld}}{\partial x} = \frac{2}{3} x\, i_1^3 + \frac{2}{3} x\, i_2^3 + i_1 i_2$$

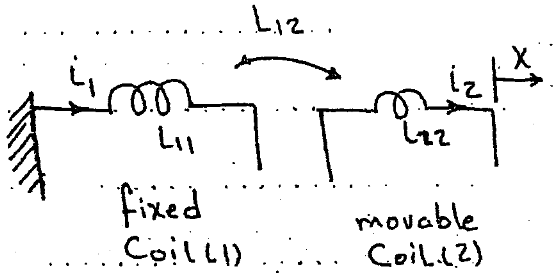
$$\therefore w_{mech} = \int_0^{0.05} F_e\, dx = \frac{x^2 i_1^3}{3} + \frac{x^2 i_2^3}{3} + i_1 i_2 x \Big|_0^{0.05} = \checkmark \text{ J}$$

$$\text{if } w_{ele} = \int_0^{0.05} \left(i_1 \frac{d\lambda_1}{dx} + i_2 \frac{d\lambda_2}{dx} \right) dx = i_1^3 x^2 + i_2^3 x^2 + 2 i_1 i_2 x \Big|_0^{0.05}$$

Sheet (1) (Cont'd)

Pb(4)

Given:



- $L_{11} = L_{22} = 3 + \frac{2}{3X}$ (mH)
- $L_{12} = L_{21} = \frac{1}{3X}$ (mH)

Required:

(a) If $i_1 = 5A$ D.C & $i_2 = 0$, find the electrical force at $X = 0.01$ m.

(b) If $i_1 = 5A$ D.C & Coil(2) is open circuited & moves in the (+ve) X-direction with constant speed 20 m/S, find the voltage across Coil(2) at $X = 0.01$ m.

Solution:

(a) $$F_e = \frac{1}{2} i_1^2 \frac{dL_{11}}{dX} + \frac{1}{2} i_2^2 \frac{dL_{22}}{dX} + i_1 \cdot i_2 \frac{dL_{12}}{dX}$$

$$\therefore \frac{dL_{11}}{dX} = \frac{-2}{3X^2} \quad (\text{mH/m})$$

(2-51)

$\therefore i_2 = 0 \quad , i_1 = 5A$

$$\boxed{\therefore F_e = \frac{1}{2}(5)^2 \left(\frac{-2}{3(0.01)^2}\right) * 10^{-3} = -83.33\,N}$$

(b) $\because$ Coil(2) is open circuited $\Rightarrow i_2 = 0$

$$\therefore e_2 = \frac{d\lambda_2}{dt} \quad \& \quad \lambda_2 = \underbrace{L_{22} i_2}_{\to 0} + L_{12} i_1$$

$$\therefore \lambda_2 = L_{12} i_1 .$$

$$\therefore e_2 = \frac{d}{dt}(i_1 L_{12}) \longrightarrow$$ but we donot have $L_{12}(t)$ to differentiate w.r.t. time.

$$\therefore e_2 = \frac{d\lambda_2}{dx} \cdot \frac{dx}{dt} \quad , \quad \frac{dx}{dt} = \text{speed} = 20\,m/s .$$

$$\therefore \frac{d\lambda_2}{dx} = i_1 \frac{dL_{12}}{dx} \quad , \quad L_{12} = \frac{1}{3x}\ (mH)$$

$$\therefore \frac{dL_{12}}{dx} = -\frac{1}{3x^2} \quad (mH/m)$$

$$\therefore e_2 = 5 * \frac{-1}{3(0.01)^2} * 10^{-3} * 20$$

$\hookrightarrow$ due to (mH)

$$\boxed{\therefore e_2 = -333.33\,V}$$

(2-52)

Pb(5) – (I)

• **Given:** for the same figure of pb(4)

$i_1 = 7.07 \sin 377t$, $i_2 = 0$, $x = 0.1\,m$

• **Required:**

(a) find the instaneous force

(b) The average force

Solution:-

(a) $$f_e = \frac{1}{2} i_1^2 \frac{dL_{11}}{dx} + \frac{1}{2} i_2^2 \frac{dL_{22}}{dx} + i_1 i_2 \frac{dM}{dx}$$ ($M \to L_{12}$)

but $i_2 = 0$ & $L_{11} = 3 + \frac{2}{3x}$ (from pb(4)).

$$\therefore f_e = \frac{1}{2}(7.07)^2 \sin^2 377t \cdot \underbrace{\left(\frac{-2}{3x^2}\right) \times 10^{-3}}_{\frac{dL_{11}}{dx}}$$

At $x = 0.1\,m$

$$\therefore \boxed{f_e = -1.67 \sin^2 377t}$$ → instantenous force.

b) $F_{e|avg} = ?$ $\therefore \sin^2 377t = \frac{1}{2}(1 - \cos(2 \times 377t))$

$$\therefore f_e = \frac{-1.67}{2}\left[1 - \cos(2 \times 377t)\right]$$

∴ The average of $\cos(2 \times 377t) = $ zero.

$$\therefore \boxed{Fe|_{avg} = \frac{-1.67}{2} = -0.835\,N}$$

. You can also use the average of $\sin^2\omega t$ or $\cos^2\omega t$ equals $(\frac{1}{2})$

$$\therefore \boxed{Fe|_{avg} = -1.67(\tfrac{1}{2}) = -0.835\,N}$$

or $\underline{F_{e\,avg} = \frac{1}{2} i_{1\,rms}^2 \frac{dL_{11}}{dx}}$; $i_{1\,rms} = \frac{7.07}{\sqrt{2}}$

Pb(5) - II

(a). If $i_1 = 10A$, $i_2 = -5A$, find the mechanical work done in increasing (X) from 0.1m to 1m.

(b). Does the force tend to increase or decrease (X)?

(c). How much energy is supplied by source of coil (1) & coil (2)?

(d). find the average force at $X = 0.5\,m$, when coil (2) is short circuited & sinusoidal voltage of 377V (rms) & 60Hz is applied to coil (1)

(2 - 54)

Soln:-

(a) $W_{mech} = \int_{0.1}^{1} f_e \, dx$

$\therefore f_e = \frac{1}{2} i_1^2 \frac{dL_{11}}{dx} + \frac{1}{2} i_2^2 \frac{dL_{22}}{dx} + i_1 i_2 \frac{dL_{12}}{dx}$ → M

$\therefore \frac{dL_{11}}{dx} = \frac{dL_{22}}{dx} = \frac{-2}{3x^2} \times 10^{-3}$ (from pb(4)).

$\therefore \frac{dL_{12}}{dx} = \frac{-1}{3x^2} \times 10^{-3}$ (from pb(4)).

At $i_1 = 10A$, $i_2 = -5A$

$\therefore f_e = \left[\frac{1}{2}(10)^2\left(\frac{-2}{3x^2}\right) + \frac{1}{2}(-5)^2\left(\frac{-2}{3x^2}\right) + (10 \times -5)\left(\frac{-1}{3x^2}\right)\right] \times 10^{-3}$

$$\therefore \boxed{f_e = \frac{-1}{40x^2}}$$

$$\therefore \boxed{W_{mech} = \int_{0.1}^{1} \frac{-1}{40x^2} \, dx = -0.225 \text{ Joule}}$$

(b) Since $f_e = \frac{-1}{40x^2}$ is always (-ve)

$\therefore$ $\boxed{f_e \text{ tends to decrease } (x)}$

(2-55)

(C) The energy supplied by source (1):

$$W_{e1} = \int_{\lambda_1}^{\lambda_2} i_1 d\lambda_1 = \int_{x_1}^{x_2} i_1 \frac{d\lambda_1}{dx} . dx.$$

$\because \lambda_1 = i_1 L_{11} + i_2 L_{12}$

$$\therefore \frac{d\lambda_1}{dx} = i_1 \frac{dL_{11}}{dx} + i_2 \frac{dL_{12}}{dx}.$$

$$\frac{dL_{11}}{dx} = \frac{2}{-3x^2} \times 10^{-3} , \quad \frac{dL_{12}}{dx} = \frac{-1}{3x^2} \times 10^{-3}.$$

$$\therefore W_{e1} = \int_{0.1}^{1} 10 \left[10 \left(\frac{2}{-3x^2} \right) + (-5)\left(-\frac{1}{3x^2} \right) \right] \times 10^{-3} \, dx.$$

$$= \int_{0.1}^{1} \frac{-0.05}{x^2} dx$$

$$\boxed{\therefore W_{e1} = -0.45 \text{ Joule.}}$$

• The energy supplied by source (2):-

$$W_{e2} = \int_{\lambda_1}^{\lambda_2} i_2 \, d\lambda_2 \quad , \quad \lambda_2 = i_2 L_{22} + i_1 L_{21}$$

$$\therefore W_{e2} = \int_{x_1}^{x_2} i_2 \frac{d\lambda_2}{dx} . dx.$$

$$\therefore \frac{d\lambda_2}{dX} = (-5)\left(\frac{-2}{3X^2}\right) + (10)\left(\frac{-1}{3X^2}\right)$$

$$w_{ele_2} = \int_{0.1}^{1} (-5)\left[\frac{10}{3X^2} - \frac{10}{3X^2}\right] dX = 0$$

d) $$F_{e_{avg}} = \frac{1}{2}\, i_{1_{rms}}^2 \frac{dL_{11}}{dX} + \frac{1}{2}\, i_{2_{rms}}^2 \frac{dL_{22}}{dX} + i_1 i_2 \cos(\theta_1 - \theta_2) \frac{dM}{dX}$$

But i_1 & i_2 are unknowns.

- We have to get i_1 & i_2 :

$$e_2 = \frac{d\lambda_2}{d\tau} = 0 \implies \lambda_2 = K \quad (\text{Take } K = 0)$$

$$\lambda_2 = L_{22} i_2 + M i_1 = 0 \implies i_2 = \frac{-M}{L_{22}} i_1 \longrightarrow (1)$$

$$e_1' = \frac{d\lambda_1}{d\tau} = 377\sqrt{2}\cos\omega\tau \quad ; \quad \omega = 2\pi(60) = 377$$

$$\lambda_1 = L_{11} i_1 + M i_2 = \frac{377\sqrt{2}}{377}\sin 377\tau + K_2 \longrightarrow (2)$$

Take $K_2 = 0$

From (1) in (2)

$$\left(L_{11} - \frac{M^2}{L_{22}}\right) i_1 = \sqrt{2}\ \sin 377t$$

$$i_1 = \frac{\sqrt{2}\ \sin 377t}{L_{11} - \frac{M^2}{L_{22}}} \longrightarrow (3)$$

$$\& \quad i_2 = \frac{-M}{L_{22}}\, i_1 \longrightarrow (4)$$

- At $X = 0.5$ m

Put: $X = 0.5$ in L_{11}, L_{22}, M

$L_{11} = 4.33$ mH

$L_{22} = 4.33$ mH

$M = 0.67$ mH

$$\left.\frac{dL_{11}}{dX}\right|_{X=0.5} = -2.67 \text{ mH/m}$$

$$\left.\frac{dL_{22}}{dX}\right|_{X=0.5} = -2.67 \text{ mH/m}$$

$$\left.\frac{dM}{dX}\right|_{X=0.5} = -1.33 \text{ mH/m}$$

$i_1 = 334.61 \sin(377t)$, $i_2 = -51.42 \sin(377t)$

But to use $F_{e_{avg}} = \frac{1}{2} i_{1_{rms}}^2 \frac{dL_{11}}{dX} + \frac{1}{2} i_{2_{rms}}^2 \frac{dL_{22}}{dX} + i_1 i_2 \cos(\theta_1 - \theta_2) \frac{dM}{dX}$

i_1 & i_2 must be in the Form

$$i_1 = I_{m_1} \sin(\omega t - \theta_1)$$

$$i_2 = I_{m_2} \sin(\omega t - \theta_2)$$

$$i_1 = 334.61 \sin(377 t)$$

$$i_2 = 51.42 \sin(377 t + \pi)$$

$$\therefore F_{e_{avg}} = \frac{1}{2} \left(\frac{334.61}{\sqrt{2}}\right)^2 * (-2.67 * 10^{-3})$$

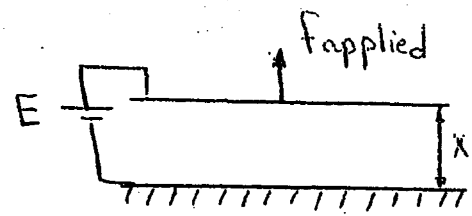
$$+ \frac{1}{2} \left(\frac{51.42}{\sqrt{2}}\right)^2 * (-2.67 * 10^{-3})$$

$$+ \left(\frac{334.61}{\sqrt{2}}\right)\left(\frac{51.42}{\sqrt{2}}\right) \cos(0 - \pi) * (-1.33 * 10^{-3})$$

$$= -65.06 \ N$$

Pb(11): Mid-Term 2013 (2-73)

Given: $C = \frac{1}{X}\ \mu F.$



- Initial pos. $\longrightarrow E = 200V,\ X = 0.01.$
- Cycle:-

(a) $E = 200V$, $X: \underset{C_1}{0.01} \longrightarrow \underset{C_2}{0.02\,m}.$

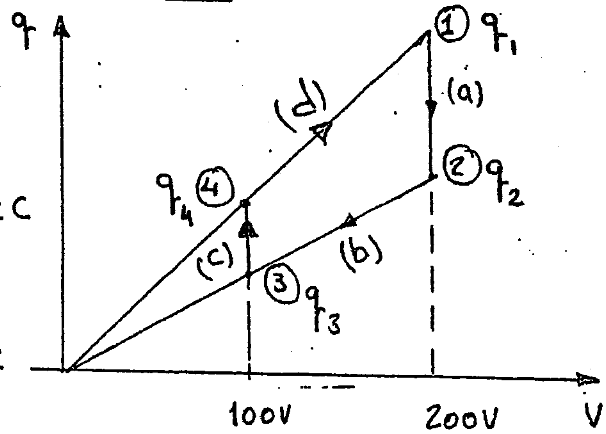
(b) $X = \underset{C_2}{0.02}$, $E: 200 \longrightarrow 100V.$

(c) $E = 100V$, $X: \underset{C_2}{0.02} \longrightarrow \underset{C_1}{0.01}.$

(d) $X = \underset{C_1}{0.01}$, $E: 100V \longrightarrow 200V.$

- Req:- ΔW_{mech}, ΔW_{elec} for the closed path

Solution:-



$\because q_1 = C_1 V$

$\therefore q_1 = 200 * \frac{1}{0.01} \times 10^{-6} = 0.02C$

$\therefore q_2 = 200 * \frac{1}{0.02} \times 10^{-6} = 0.01C$

$\therefore q_3 = 100 * \frac{10^{-6}}{0.02} = 0.005C.$

$\therefore q_4 = 100 \frac{\times 10^{-6}}{0.01} = 0.01C$

- Considering Pathes:- (2-74)

(a) $V = 200V = \text{Const}.$

$q_1 = 0.02 \longrightarrow q_2 = 0.01.$

$$\therefore \Delta W_{elec} = \int_{q_1}^{q_2} V\,dq = V(q_2 - q_1) = -2J.$$

(Area btn the path of motion & the vertical axis will give (−2J) as the motion from 1 → 2 Causes decrease in charge ∴ dq = −ve).

$\therefore \Delta W_{mech} =$ Area. btn 2 C/cs & path of motion but with −ve value as the path of motion is 1 → 2.

or $\Delta W_{mech} = \Delta W_{elec} - \Delta W_{fld}$

$$\Delta W_{fld} = W_{fld}\Big|_{2} - W_{fld}\Big|_{1}$$

$$= \frac{1}{2} C_2 V_2^2 - \frac{1}{2} C_1 V_1^2 \quad , (V_1 = V_2 = 200V)$$

$$= \frac{1}{2}(200)^2 \left[\frac{1}{0.02} - \frac{1}{0.01}\right] \times 10^{-6} = -1J.$$

$$\therefore \Delta W_{mech} = -2 - (-1) = -1J.$$

or Directly:

$$\Delta W_{mech} \left[\frac{1}{2}(200)(0.01) - \frac{1}{2}(200)(0.02)\right] = -1J$$

(2-75)

(b)

$$\Delta W_{mech} = 0 \quad (x = \text{Const}).$$

$$\Delta W_{elec.} = \int_{q_2}^{q_3} V\, dq.$$

or $\Delta W_{elec} = \Delta W_{fld} = W_{fld}|_{3} - W_{fld}|_{2}$

$$= \frac{1}{2} V_3 q_3 - \frac{1}{2} q_2 V_2$$

$$= \frac{1}{2}(100 * 0.005) - \frac{1}{2}(200 * 0.01)$$

$$= -0.75\, J.$$

or directly: $\Delta W_{elec} = -(100+200) * \frac{1}{2}(0.01 - 0.005) = -0.75 J$

(c)

$$\Delta W_{elec} = \int_{3}^{4} V\, dq. = \int_{q_3}^{q_4} 100\, dq = 100(0.01 - 0.005)$$

$$\therefore \Delta W_{elec} = 0.5 \text{ Joule}.$$

ΔW_{mech} = Area btn 2 C/Cs but with +ve value.

$$= \frac{1}{2}(100)(0.01 - 0.005) = 0.25\, J$$

$$\boxed{\therefore \Delta W_{elec} = 0.5 \text{ Joule} \;\&\; \Delta W_{mech}\; 0.25 J}$$

(2-76)

(d)

$\Delta W_{mech} = 0.$

$\Delta W_{elec} = \Delta W_{fld}.$

$$= W_{fld}\Big|_{①} - W_{fld}\Big|_{④}.$$

$$= \frac{1}{2}\underbrace{(200)}_{v_1}\underbrace{(0.02)}_{q_1} - \frac{1}{2}\underbrace{(100)}_{v_4}\underbrace{(0.01)}_{q_4}$$

$= \underline{\underline{1.5 \text{ Joule.}}}$

$$\therefore \Delta W_{elec}\Big|_{Total} = \Delta W_{elec}\Big|_a + \Delta W_{elec}\Big|_b + \Delta W_{elec}\Big|_c + \Delta W_{elec}\Big|_d$$

$$\boxed{\Delta W_{elec} = -0.75 \text{ J.}}$$

∴ the energy is supplied to the battery (-ve value).

&

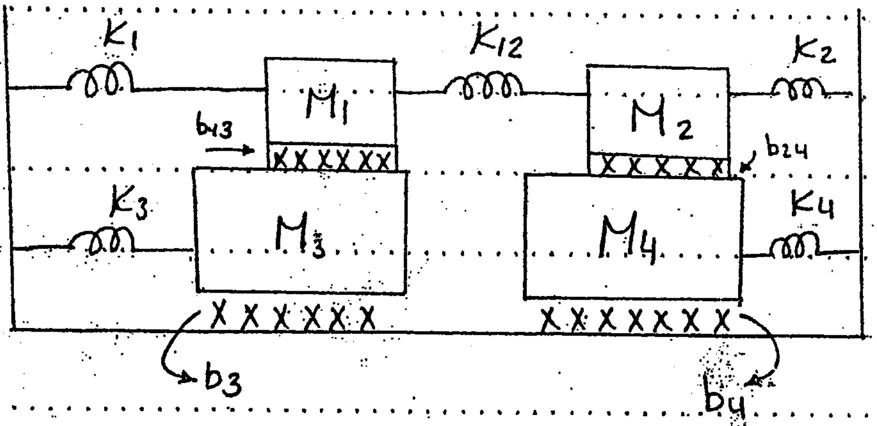
$$\Delta W_{mech} = \Delta W_{mech}\Big|_a + \Delta W_{mech}\Big|_b + \Delta W_{mech}\Big|_c + \Delta W_{mech}\Big|_d = \underline{\underline{-0.75 \text{ J.}}}$$

∴ work is done by external force (-ve value).

Sheet (3)

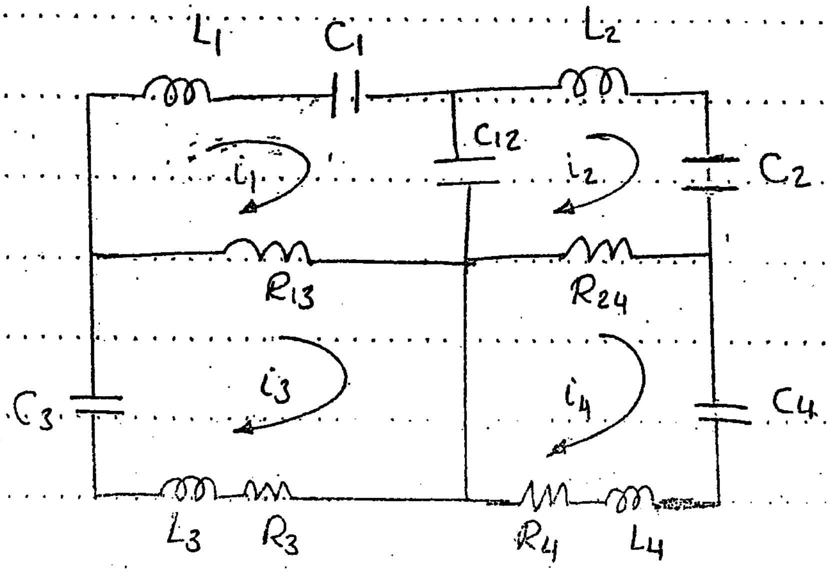
Pb (3):

(a) Required:- obtain the analogs for the mechanical system (i.e loop & nodal based circuits)



for loop based circuit :-

No. of masses = No. of loops = 4.



(3-19)

Where:-

- $M_1 \rightarrow L_1$, $M_2 \rightarrow L_2$, $M_3 \rightarrow L_3$, $M_4 \rightarrow L_4$
- $K_1 \rightarrow C_1$, $K_2 \rightarrow C_2$, $K_3 \rightarrow C_3$, $K_4 \rightarrow C_4$
- $K_{12} \rightarrow C_{12}$
- $B_3 \rightarrow R_3$, $B_4 \rightarrow R_4$, $B_{12} \rightarrow R_{13}$, B_{24} R_{24}
- $\dot{X}_1 \rightarrow i_1$, $\dot{X}_2 \rightarrow i_2$, $\dot{X}_3 \rightarrow i_3$, $\dot{X}_4 \rightarrow i_4$

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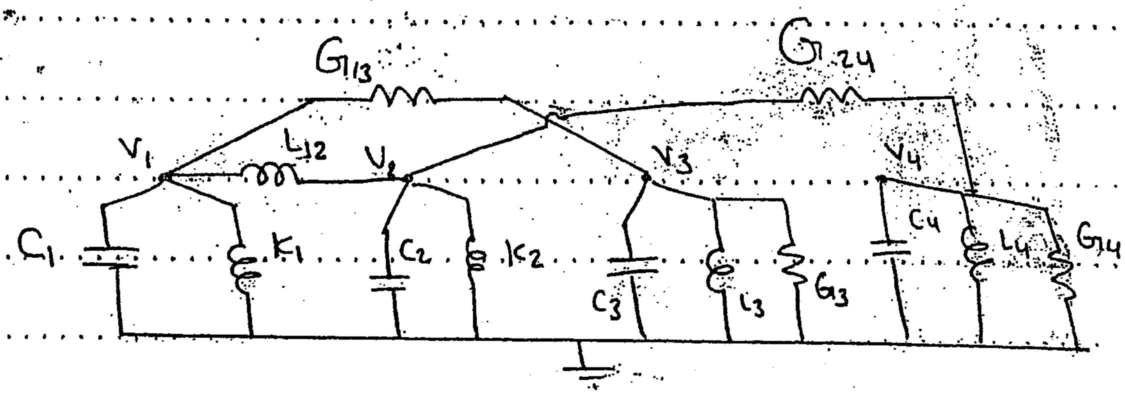
for Node based circuit:-

No. of masses = No. of nodes = 4.

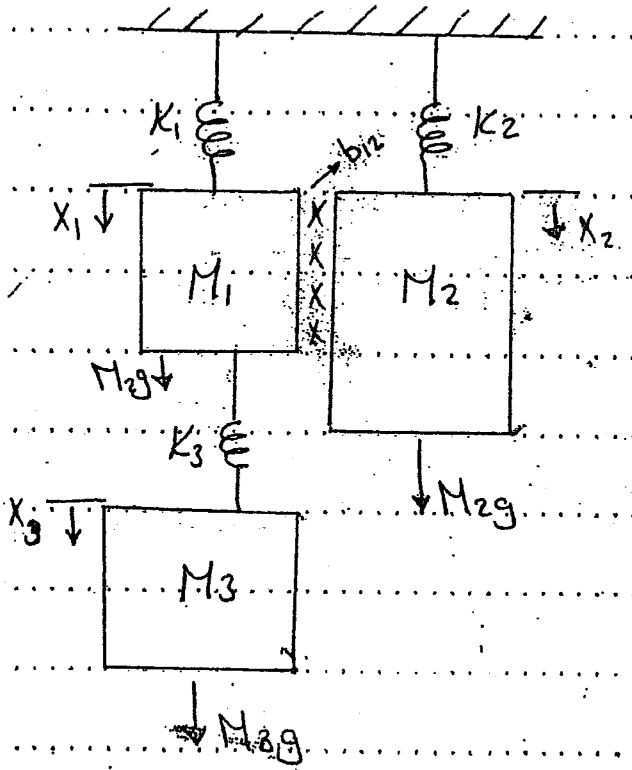


Where:

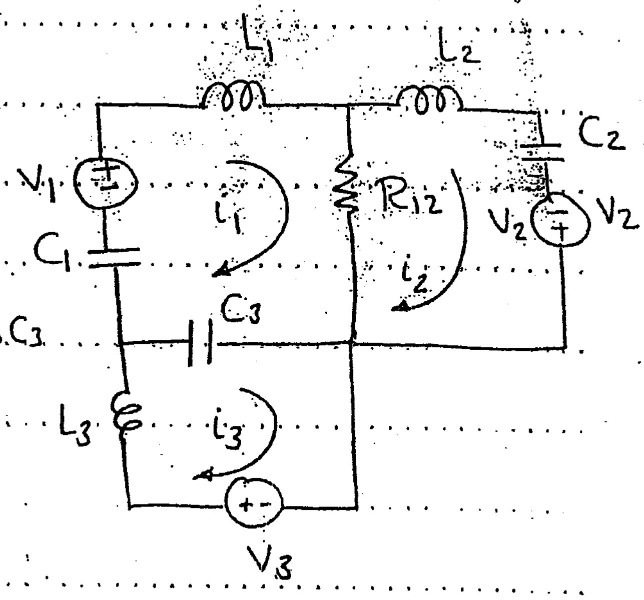
- $M_1 \rightarrow C_1$, $M_2 \rightarrow C_2$, $M_3 \rightarrow C_3$, $M_4 \rightarrow C_4$
- $K_1 \rightarrow L_1$, $K_2 \rightarrow L_2$, $K_3 \rightarrow L_3$, $K_4 \rightarrow L_4$
- $B_3 \rightarrow G_3$, $B_4 \rightarrow G_4$, $B_{24} \rightarrow G_{24}$, $B_{13} \rightarrow G_{13}$
- $K_{12} \rightarrow L_{12}$
- $\dot{X}_1 \rightarrow V_1$, $\dot{X}_2 \rightarrow V_2$, $\dot{X}_3 \rightarrow V_3$, $\dot{X}_4 \rightarrow V_4$

(3-20)

P.b(3)_b



for loop based Circuit:

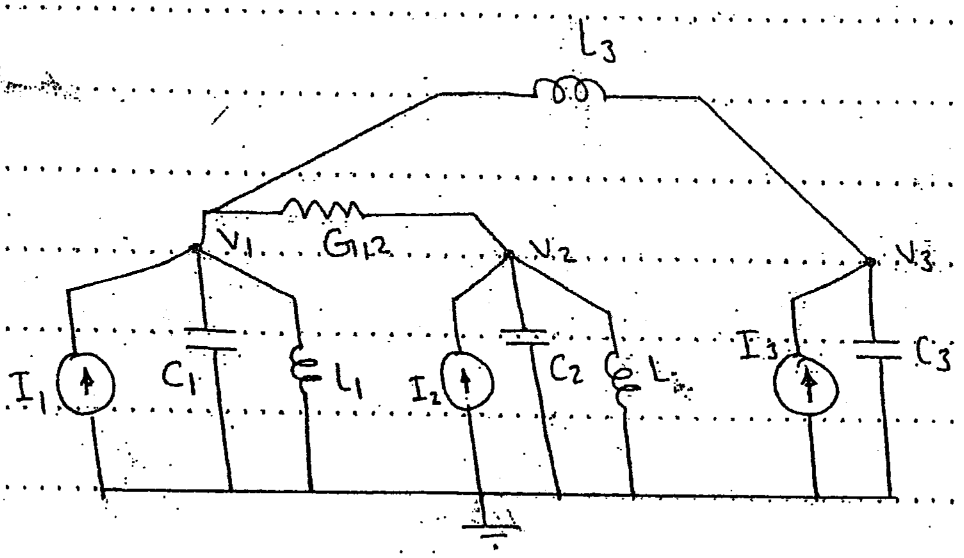


- No. of loops = 3
- $M_1 \longrightarrow L_1$, $M_2 \longrightarrow L_2$
- $M_3 \longrightarrow L_3$
- $K_1 \longrightarrow C_1$, $K_2 \longrightarrow C_2$, $K_3 \longrightarrow C_3$
- $b_{12} \longrightarrow R_{12}$
- $M_1 g \longrightarrow V_1$, $M_2 g \longrightarrow V_2$
- $M_3 g \longrightarrow V_3$
- $\dot{X}_1 \longrightarrow i_1$, $\dot{X}_2 \longrightarrow i_2$, $\dot{X}_3 \longrightarrow i_3$

(3-21)

for Node based Circuit:-

No of nodes = 3

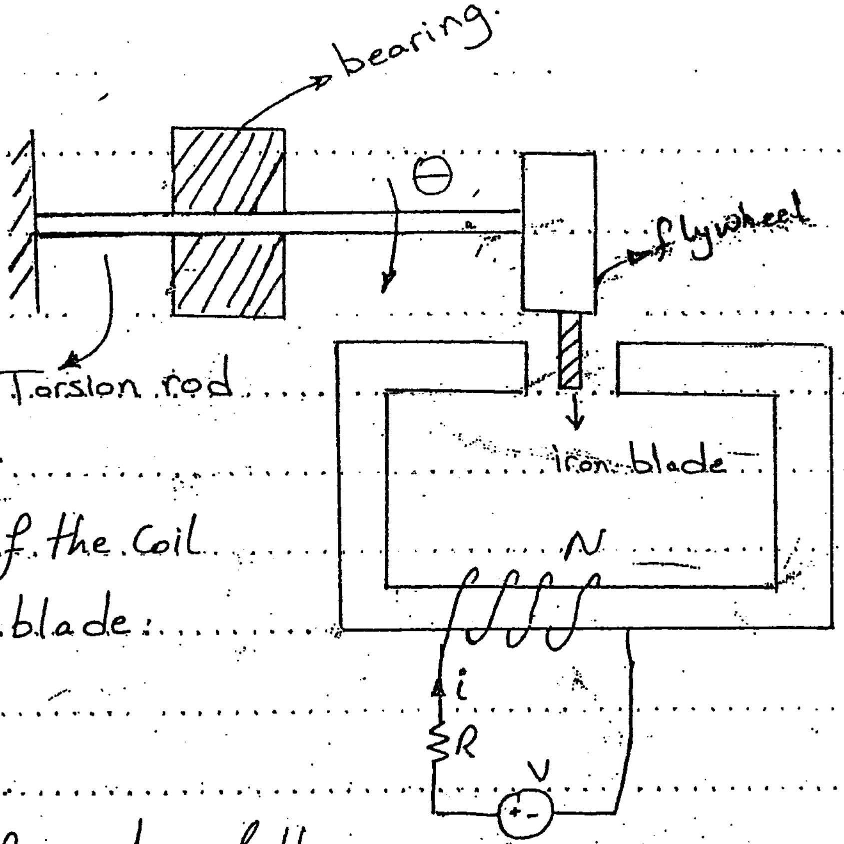


- $C_1 \longrightarrow M_1$, $C_2 \longrightarrow M_2$, $C_3 \longrightarrow M_3$
- $K_1 \longrightarrow L_1$, $K_2 \longrightarrow L_2$, $K_3 \longrightarrow L_3$
- $b_{12} \longrightarrow G_{12}$
- $M_1 g \longrightarrow I_1$, $M_2 g \longrightarrow I_2$, $M_3 g \longrightarrow I_3$
- $\dot{x}_1 \longrightarrow V_1$, $\dot{x}_2 \longrightarrow V_2$, $\dot{x}_3 \longrightarrow V_3$

(3-80)

Example (2): Page (104)

Given:



- Inductance of the coil due to the iron blade:

$$L = A + B\theta$$

- J : moment of inertia of the rotating parts
- b ≡ friction coefficient
- K : stifness constant of the torsion rod

Required:

(a) write equation of motion

(b) Linearize the eqns & identify the non-Linear terms.

(3-41)

Solution:-

(a)

[1] $\therefore L = A + B\theta$

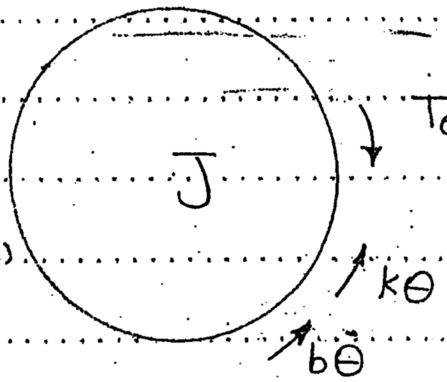
$$\therefore \frac{dL}{d\theta} = B \quad , \quad \frac{d^2L}{d\theta^2} = 0$$

[2] $T_e = \frac{1}{2} I^2 \frac{dL}{d\theta} = \frac{1}{2} I^2 B$

[3] Mechanical eqns:-

$$\therefore J\ddot{\theta} = T_e - b\dot{\theta} - k\theta$$

$$\boxed{\therefore \frac{1}{2} I^2 B = J\ddot{\theta} + b\dot{\theta} + k\theta} \rightarrow (1)$$



[4] Electrical eqns:-

$$V = iR + \frac{d\lambda}{dt} \quad , \quad \frac{d\lambda}{dt} = i \frac{dL}{d\theta}\frac{d\theta}{dt} + L\frac{di}{dt}$$

$$\therefore V = IR + BI\frac{d\theta}{dt} + (A + B\theta)\frac{dI}{dt}$$

$$\boxed{\therefore V = IR + BI\frac{d\theta}{dt} + A\frac{dI}{dt} + B\theta\frac{dI}{dt}} \rightarrow (2)$$

(3-42)

(b) from (1) & (2), the non-Linear terms are:-

$$\boxed{\frac{1}{2} I^2 B \ , \ B I \frac{d\theta}{dt} \ , \ B \theta \frac{dI}{dt}}$$

Linearization:-

Let: $\Theta = \Theta_o + \theta$, $V = V_o + v$, $I = I_o + i$

At steady-state:-

At eqn (1): $\ddot{\theta} = 0$, $\dot{\theta} = 0$

$$\therefore \boxed{\frac{1}{2} I_o^2 B = K \theta_o}$$

At eqn (2):-

$$\dot{\theta} = 0 \ , \ \frac{dI}{dt} = 0$$

$$\therefore \boxed{V_o = I_o R}$$

Now: substituting in (1) & (2) with: $\Theta = \Theta_o + \theta$, $V = V_o + v$ & $I = I_o + i$

→ from eqn (1):-

$$\therefore \frac{1}{2} (I_o + i)^2 B = J \ddot{\theta} + b \dot{\theta} + K(\theta_o + \theta)$$

$$\therefore \frac{1}{2} (\cancel{I_o^2} + 2 I_o i + i^2) B = J \ddot{\theta} + b \dot{\theta} + \cancel{K \theta_o} + K \theta$$

(3-43)

- Using $K\theta_0 = \frac{1}{2} I_0^2 B$

$$\therefore \boxed{I_0 i B = J\ddot{\theta} + b\dot{\theta} + K\theta} \longrightarrow (3)$$

→ from eqn (2):

$$\therefore \cancel{V_0} + v = (\cancel{I_0} + i)R + B(I_0 + i)\frac{d\theta}{dt} + A\left(\frac{di}{dt}\right) + B(\theta_0 + \theta)\frac{di}{dt}$$

- Using $V_0 = I_0 R$

$$\therefore v = iR + I_0 B\frac{d\theta}{dt} + \underbrace{i\frac{d\theta}{dt}}_{\simeq 0} + A\frac{di}{dt} + B\theta_0\frac{di}{dt} + \underbrace{B\theta\frac{di}{dt}}_{\simeq 0}$$

$$\therefore \boxed{v = iR + I_0 B\frac{d\theta}{dt} + (A + B\theta_0)\frac{di}{dt}} \longrightarrow (4)$$

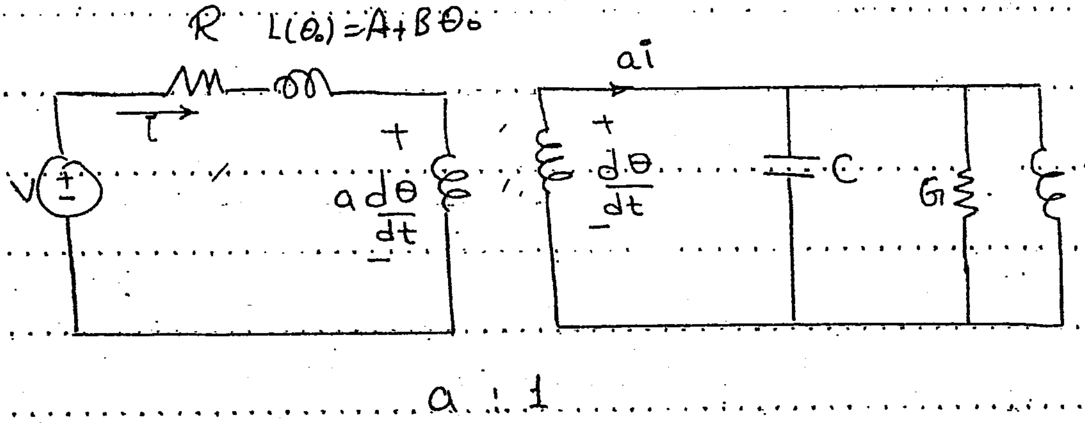
- To get the equivalent electrical system:

$\therefore$ let $I_0 B = a$ in eqn (3) & (4)

$$\therefore \boxed{\begin{aligned} a i &= J\ddot{\theta} + b\dot{\theta} + K\theta \\ \& \\ v &= iR + (A + B\theta_0)\frac{di}{dt} + a\frac{d\theta}{dt} \end{aligned}}$$

(3-46)

Like the previous example, the system can be modeled using transformer:

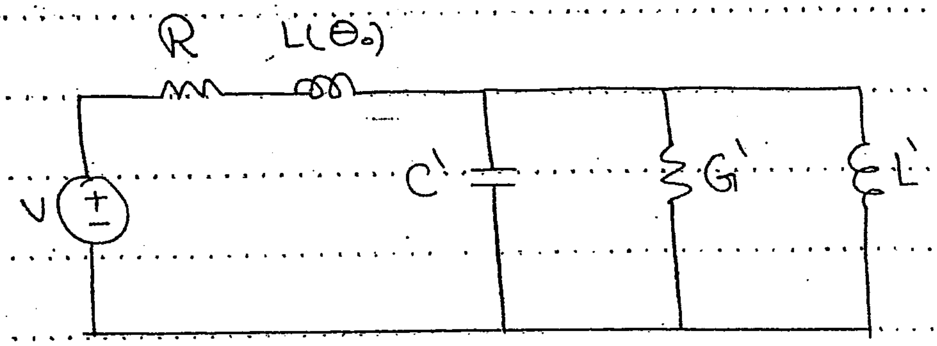


$C \equiv J$

$L \equiv K$

$G \equiv b$

The referred to primary circuit:-



$C' = \frac{C}{a^2}$

$L' = a^2 L$

$G' = \frac{G}{a^2}$

(3-50)

Pb(4): (Mid-term 2011).

• Given: • Cylindrical electro magnet

• $a = 2mm$, $c = 40mm$

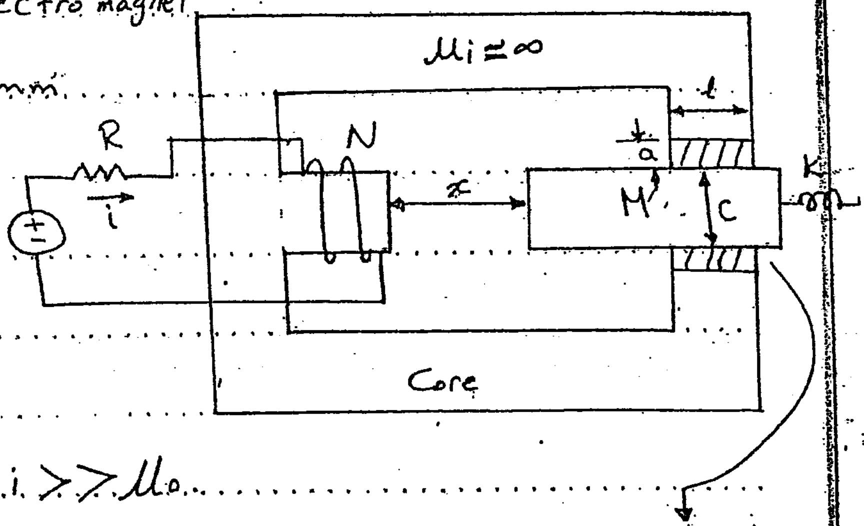
• $l = 40mm$

• $R = 3.5\Omega$

• $V = 110\ V(rms)$

• $f = 60Hz$

• $N = 500$, $\mu_i >> \mu_o$



• Required:

At $X = 5mm$

(a) The max. air gap flux density.

(b) The average value of the electrical force.

• Solution:

(a)

• $B_{air-gap} = B_g = \dfrac{\Phi_g}{A} \Rightarrow \Phi_g = ?\ ,\ A = ?$

• The system is cylinderical $\Rightarrow$ The mass (M) has circular cross section area:

$$\therefore A = \frac{\pi}{4} c^2 = \frac{\pi}{4} (40 \times 10^{-3})^2 = 1.256 \times 10^{-3}\ m^2$$

(3-51)

To get Φ_g:

$\because \mu_i >> \mu_r \Rightarrow$ linear system

$\therefore NI = \Phi R \Rightarrow \Phi = \frac{NI}{R}$

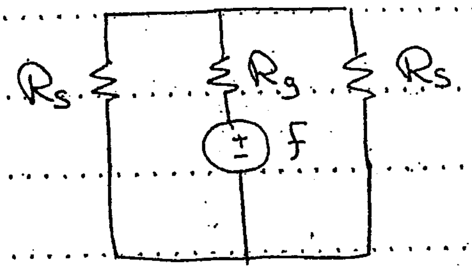
$\therefore$ we have to get I, R

To get R:

The equivalent circuit will be:

R_s: sleeve rel.

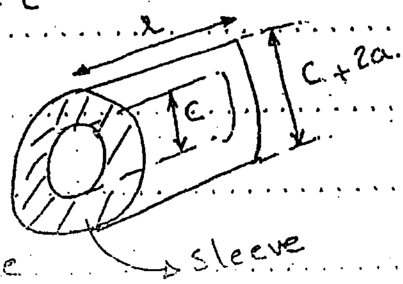
R_g: air gap rel.



$\therefore R = R_{eq} = \frac{R_s}{2} + R_g \Rightarrow R_s, R_g = ?$

$\therefore R_g = \frac{x}{\mu_o A} = \frac{x}{\mu_o \frac{\pi}{4} C^2} \longrightarrow$ o.k.

$\therefore R_s = \frac{a}{\mu_o A_s}$

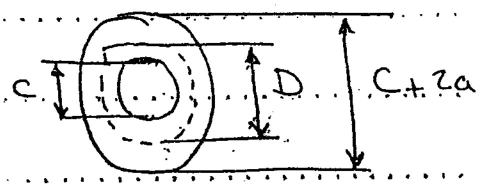


$\therefore A_s$ is non-uniform as the system is cylinderical.

$\therefore$ we have to take the average area.

N.B : The area of the sleeve will be the lateral surface area (المساحة الجانبية) not the cross section area (Due to the flux path of motion)

$$\therefore A_s = (\pi . D . l) * \frac{1}{2}$$



$$\therefore D = \frac{C + C + 2a}{2} = \frac{2C + 2a}{2} = C + a$$

N.B : we multiplied by $(\frac{1}{2})$, As the flux will cross half the area only for upper sleeve & the other half is for the lower sleeve.

$$\therefore A_s = \frac{\pi}{2} (C + a) l$$

$$\boxed{\therefore R_s = \frac{2a}{\mu_0 \pi (C+a) l}}$$

Now:

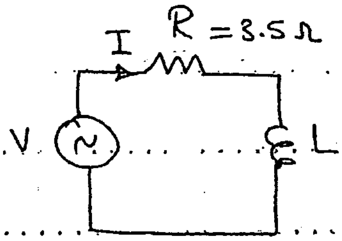
$$R = R_{eq} = R_g + \frac{R_s}{2} = \frac{5 \times 10^{-3}}{\mu_0 \frac{\pi}{4} C^2} + \frac{a}{\mu_0 \pi (C+a) l}$$

Put $a = 2\,mm$, $C = 40\,mm$, $l = 40\,mm$.

$$\boxed{\therefore R = 3467838\ AT/wb}$$

To get (I):

for A.C voltage:



$v = 110\sqrt{2}\ \sin \omega t$, $\omega = 2\pi f$

$\therefore \omega = 377$

$\therefore v = 110\sqrt{2}\ \sin 377t$

$\therefore I(t) = \frac{V}{|Z|} \sin(377 - \phi)$, $\phi = \tan^{-1} \frac{\omega L}{R}$

& $|Z| = \sqrt{R^2 + (\omega L)^2}$, $L = ??$

$\therefore L = \frac{N^2}{\mathcal{R}}$, $N = 500$, $\mathcal{R} = 3467838$ A.T/wb.

$$\boxed{\therefore L = 0.0721 \text{ Henry}}$$

$\therefore |Z| = 27.4\Omega$, $\phi = 82.66°$

$\therefore I(t) = \frac{110\sqrt{2}}{27.4} \sin(377t - 82.66°)$

$$\boxed{\therefore I(t) = 5.67 \sin(377t - 82.66°)}$$

(3-54)

$\therefore$ for max flux density $\Rightarrow I = I_{max} = 5.67$

$$\therefore N I_{max} = \Phi_{max} \cdot R$$

$$\boxed{\therefore \Phi_{max} = \frac{(500)(5.67)}{3467838} = 8.18 \times 10^{-4} \text{ wb}}$$

$$\boxed{\therefore B_{g_{max}} = \frac{\Phi_{max}}{A} = \frac{8.14 \times 10^{-4}}{\frac{\pi}{4} c^2} = 0.651 \text{ Tesla}}$$

(b) $f_e|_{avg} = ?$

$$\therefore f_e = \frac{1}{2} I^2 \frac{dL}{dx} \quad , \quad \frac{dL}{dx} = ?$$

$$L(x) = \frac{N^2}{R(x)} \quad , \quad R(x) = R = R_g + \frac{R_s}{2}$$

$$\therefore R(x) = \frac{x}{\frac{\pi}{4} c^2} + \frac{a}{\mu_o \pi (c+a) \ell}$$

Put $c = 40 \times 10^{-3}$ m, $a = 2 \times 10^{-3}$ m, $\ell = 40 \times 10^{-3}$ m.

$$\boxed{\therefore R(x) = 6.33 \times 10^{8} \, x + 301551.14}$$

$$\boxed{\therefore L(x) = \frac{(500)^2}{6.33 \times 10^{8} \, x + 301551.14}}$$

(3-55)

$$\frac{dL(x)}{dx} = \frac{-(500)^2 * 6.33\times10^8}{(6.33\times10^8\, x + 301551.14)^2}$$

At $\underline{X = 5mm}$

$$\boxed{\frac{dL}{dx} = -13.16 \ \ H/m}$$

$$\therefore Fe = \frac{1}{2}(5.67)^2 \sin^2(377t - 82.66°)(-13.16)$$

$$\boxed{\therefore Fe = -212.134 \sin^2(377 - 82.66°)}$$

$\therefore$ The average of $\sin^2$ or $\cos^2 = \frac{1}{2}$

$$\boxed{\therefore fe|_{avg} = 0.5(-212.134) = -106.07 N}$$

or directly $F_e|_{avg} = \frac{1}{2} I_{rms}^2 \frac{dL}{dx}$

(3-59)

Pb(7):

Given:

- $M = 0.1\,Kg$, $K = 22.5\,KN/m$, $B = 0$
- natural length of spring $= 25mm$ @ $i = 0$
- $B_{gap} = 0.65 \sin 377t$ Tesla
- The same system of pb(4).

Required:

- write the dynamic eqns
- If the eqns are non-linear, Linearize them

Solution:

$\boxed{1}$ Electrical eqns:

$$\therefore V = iR + \frac{d\lambda}{dt} \quad , \lambda = N\phi \; , \; \phi = B_g \cdot A$$

$$\therefore V = iR + N\frac{d}{dt}(0.65\,A \sin 377t) \quad , \text{but } i = ??$$

$$\therefore Li = \lambda = N\phi \Rightarrow i = \frac{\lambda}{L} = \frac{N\phi}{L}$$

$$\therefore L = \frac{25 \times 10^4}{6.33 \times 10^8 X + 301551}$$

(3-60)

$$\therefore i = \frac{(500)(0.65 \sin 377t)(\overset{\frac{\pi}{4}c^2}{A})(6.33\times10^8 X + 301551)}{25\times10^4}$$

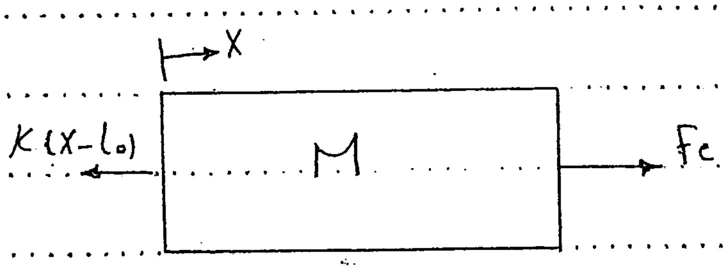
$$\therefore V = \frac{(500)(0.65)(\frac{\pi}{4})c^2 \sin 377t\,(6.33\times10^8 + 301551)}{25\times10^4} * 3.5$$

$$+ 500(0.65)\frac{d}{dt}\sin 377t$$

$$\boxed{\therefore V = (3619 \sin 377t)X + 1.724 \sin 377t + 153.9 \cos 377t}$$

$$\boxed{\text{This equation is Linear.}}$$

2 Mechanical eqn:



$$\therefore M\ddot{X} = Fe - K(X - l_0)$$

$$\therefore M\ddot{X} + K(X - l_0) = Fe.$$

(3-61)

Now:

$$F_e = \frac{1}{2} I^2 \frac{dL}{dx} = -\frac{1}{2} \phi^2 \frac{dR}{dx}$$

→ we will use $-\frac{1}{2} \phi^2 \frac{dR}{dx}$, as we have ϕ

from P.b(4): $\frac{dR}{dx} = 6.33 \times 10^8$

$$\therefore F_e = -\frac{1}{2}\left(0.65 \sin 377t \cdot \frac{\pi}{4} c^2\right)^2 \cdot 6.33 \times 10^8$$

$$\therefore F_e = -211.2 \sin^2 377t = M\ddot{X} + K(X - l_0)$$

$\because M = 0.1$ kg, $K = 22.5 \times 10^3$ N/m, $l_0 = 25 \times 10^{-3}$ m

$$\boxed{\therefore F_e = -211.2 \sin^2 377t = 0.1\ddot{X} + 22.5 \times 10^3 X - 562.5}$$

$\boxed{\therefore \text{The eqn is Linear Also}}$